Physical Parameters Required for a Habitable Dyson Sphere

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Abstract

Study of the concept of Dyson Sphere presents an opportunity to combine the areas of analytic geometry, gravitation, and thermal physics in order to find the basic physical properties of such a structure. Density, thermal conductivity, overall radius, and thickness of a sphere, combined with the physical properties of the host star give a clue to what astronomers hunting for such objects might look for.
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1 Introduction

In 1960, British physicist Freeman Dyson postulated that a solid sphere could be constructed around a star. The advantage of such a structure would be that all of the star's radiant energy could be harnessed while providing a surface area orders of magnitude greater than a planet's. Any civilization advanced enough to build such a structure would have practically unlimited amounts of power and be able to accommodate many more people than an Earth-sized world. These spheres would provide a way of detecting extremely advanced civilizations [3, p. 2075].

The exact parameters of a Dyson sphere would depend on the size and temperature of the star, the desired gravitational acceleration and temperature on the exterior surface, and the materials the sphere is made of [2].

2 Findings

The design of a Dyson Sphere would depend on several factors: the star's luminosity, the sphere's material, and the sphere's dimensions.

$$L = 4\pi R_{star}^2 \sigma T_{eff}^4 \tag{1}$$

The luminosity of a star is given by the Stefan-Boltzmann equation (1), where R_{star} is the radius of the star, T_{eff} is the effective temperature at the surface of the star, and σ is the Stefan-Boltzmann constant. T_{eff} is an approximation that is used because a star is not a perfect blackbody [1, p. 70].

More generally, the Stefan-Boltzmann equation can be represented to reflect heat flowing to cooler surroundings.

$$L = 4\pi R_{star}^2 \epsilon\sigma \left(T_{eff}^4 - T_{inner}^4\right) \tag{2}$$

In the case of Equation (2), the emissivity of the star, ϵ, is assumed to be 1 and T_{inner} denotes the temperature of the surroundings or the temperature at the inner surface of the sphere.

2.1 Mass of the sphere

The gravitational acceleration at the outer suface of the sphere is found by Newton's law of universal gravitation (3), where both the mass of the star and the mass of the spherical shall are taken into account.

$$g_0 = \frac{GM_{total}}{r_{out}^2} = \frac{G}{r_{out}^2}\left(M_{star} + M_{sphere}\right) = 9.81\frac{m}{s^2} \tag{3}$$

Because the sphere is hollow, the volume of the cavity is subtracted from the overall volume of the sphere (4). The density (mass per unit volume) of the shell's material is expressed by ρ.

$$g_0 = \frac{G}{r_{out}^2}\left(M_{star} + \frac{4}{3}\pi\rho\left(r_{out}^3 - r_{in}^3\right)\right) \tag{4}$$

2.2 Thickness of the sphere

$$\dot{Q} = L = 4\pi k \left(T_{inner} - T_{outer}\right) \frac{r_{outer} r_{inner}}{r_{outer} - r_{inner}} \tag{5}$$

$$L = \epsilon \sigma A_{star} T_{eff}^4 = 4\pi\sigma R^2 T_{eff}^4 \tag{6}$$

Equation (6) makes the assumption that the emissivity (ϵ) is 1 and that the star is spherical.

$$4\pi\sigma R^2 T_{eff}^4 = 4\pi k \left(T_{in} - T_{out}\right) \frac{r_{out} r_{in}}{r_{out} - r_{in}} \tag{7}$$

In Equation (7), Equations (5) and (6) are combined to create an expression of the radius and thickness of the sphere that is dependent on the star's temperature and radius and the desired external temperature, 298 Kelvins.

$$\frac{r_{out} - r_{in}}{r_{out} r_{in}} = \frac{k\left(T_{in} - T_{out}\right)}{\sigma R^2 T_{eff}^4} = \frac{k\left(T_{eff} - 298\mathrm{K}\right)}{\sigma R^2 T_{eff}^4} \tag{8}$$

$$\left(\frac{r_{out} - r_{in}}{r_{out} r_{in}}\right)^3 = \left(\frac{k\left(T_{eff} - 298\mathrm{K}\right)}{\sigma R^2 T_{eff}^4}\right)^3 \tag{9}$$

Because the term $\left(r_{out}^3 - r_{in}^3\right)$ appears in Equation (4), the next step was to express that term in terms of the star's characteristics.

$$\left(r_{out}^3 - r_{in}^3\right) = \left(\frac{k\left(T_{eff} - 298\mathrm{K}\right)}{\sigma R^2 T_{eff}^4}\right)^3 r_{in}^3 r_{out}^3 + 3 r_{in} r_{out} \left(r_{out} - r_{in}\right) \tag{10}$$

2.3 Combined expression

The resulting expression (11) shows the relationship between the thermal conductivity, k, material density ρ, and the inner and outer radii, r_{in} and r_{out}.

$$\frac{g_0}{G} = \frac{M_{star} + \frac{4}{3}\pi\rho\left[\left(\frac{k(T_{eff} - 298\mathrm{K})}{\sigma R^2 T_{eff}^4}\right)^3 r_{in}^3 r_{out}^3 + 3 r_{in} r_{out}\left(r_{out} - r_{in}\right)\right]}{r_{out}^2} \tag{11}$$

All of this is dependent on three characteristics of the star being enclosed. The stellar mass, M_{star}, stellar radius, R, and effective surface temperature of the star, T_{eff}.

2.4 Special cases

In the case where the inner surface of the Dyson Sphere is at the star's surface, but not below it, $r_{in} = R$.

$$\frac{g_0}{G} = \frac{M_{star} + \frac{4}{3}\pi\rho\left[\left(\frac{k(T_{eff} - 298\mathrm{K})}{\sigma R T_{eff}^4}\right)^3 r_{out}^3 + 3 R r_{out}\left(r_{out} - R\right)\right]}{r_{out}^2} \tag{12}$$

In the case where the shell is infinitesimally thin, $r_{in} = r_{out}$, the radius of the shell can be found with this expression.

$$\frac{g_0}{G} = \frac{M_{star} + \frac{4}{3}\pi\rho\left[\left(\frac{k(T_{eff} - 298\mathrm{K})}{\sigma R^2 T_{eff}^4}\right)^3 r_{out}^6\right]}{r_{out}^2} \tag{13}$$

In the (highly unlikely) case where the star's $T_{eff} = 298\mathrm{K}$ and the interior temperature is the same as the desired exterior temperature, the star's luminosity and the sphere's thermal conductivity drop away.

$$\frac{g_0}{G} = \frac{M_{star} + \frac{4}{3}\pi\rho\left[3 r_{in} r_{out}\left(r_{out} - r_{in}\right)\right]}{r_{out}^2} \tag{14}$$

2.5 Variations on the inner radius

In Table 1, the physical parameters for the star described in the "Conditions" subsection of this report are taken, along with the density ρ and thermal conductivity, k of iron. Then the outer radius r_{out} was varied, producing different values of the shell's inner radius, thickness, and volume of material required to construct it.

Table 1: Variations on outer shell radius

ρ ($\frac{kg}{m^3}$)	k ($\frac{W}{mK}$)	r_{in} (AU)	r_{in} (m)	r_{out} (AU)	r_{out} (m)	shell thickness (m)	material req. (m^3)
7874	80.4	0.05	7.48E+09	0.05	7.48E+09	1.00E+05	7.03E+25
7874	80.4	0.10	1.50E+10	0.10	1.50E+10	1.00E+05	2.81E+26
7874	80.4	0.15	2.24E+10	0.15	2.24E+10	5.00E+06	3.16E+28
7874	80.4	0.20	2.99E+10	0.20	2.99E+10	4.20E+06	4.73E+28
7874	80.4	0.25	3.74E+10	0.25	3.74E+10	2.50E+06	4.39E+28
7874	80.4	0.30	4.49E+10	0.30	4.49E+10	1.60E+07	4.05E+29
7874	80.4	0.35	5.24E+10	0.35	5.24E+10	4.80E+06	1.65E+29
7874	80.4	0.40	5.98E+10	0.40	5.98E+10	7.10E+06	3.19E+29
7874	80.4	0.45	6.73E+10	0.45	6.73E+10	6.70E+06	3.82E+29
7874	80.4	0.50	7.48E+10	0.50	7.48E+10	2.17E+07	1.53E+30
7874	80.4	0.55	8.23E+10	0.55	8.23E+10	1.34E+07	1.14E+30
7874	80.4	0.60	8.98E+10	0.60	8.98E+10	8.80E+06	8.91E+29
7874	80.4	0.65	9.73E+10	0.65	9.72E+10	1.75E+07	2.08E+30
7874	80.4	0.70	1.05E+11	0.70	1.05E+11	1.00E+07	1.38E+30
7874	80.4	0.75	1.12E+11	0.75	1.12E+11	1.00E+07	1.58E+30
7874	80.4	0.80	1.20E+11	0.80	1.20E+11	1.40E+07	2.52E+30

$$\left(\frac{k\left(T_{eff}-298\right)}{\sigma R^2 T_{eff}} r_{out}\right)^3 r_{in}^3 - \left(3r_{out}\right) r_{in}^2 + \left(3r_{out}^2\right) r_{in} - \frac{3}{4}\left(\frac{\frac{g_o}{G} r_{out}^2 - M_{star}}{\pi\rho}\right) = 0 \tag{15}$$

The initial expression (11) was reworked to provide a cubic expression (15), where r_{in} is the variable that needs to be solved for and the others are all treated as constants.

3 Setup

3.1 Assumptions

To simplify this thought experiment, it is important to make some assumptions. The thermal conductivity, k, of the sphere is assumed to be uniform, but is not known. Any atmosphere clinging to the outside of the sphere is assumed to be insignificant.

The amount of energy leaving the star is taken to be equivalent to the energy striking the interior surface of the sphere, albeit with a lower flux. This is why Equations (5) and (6) could be taken to be equivalent.

The shell and star are assumed to be perfectly symmetric spheres with identical barycenters, thus preventing any possible variances in heat or gravity on the outer surface of the shell.

3.2 Conditions

Because a Dyson Sphere would presumably be built from different materials, its density and thermal conductivity would not be known exactly, but must still be taken into account. The distribution of the differing materials is presumed to be homogenous, such that there not areas of denser material or areas where heat transfers through more easily.

The star used in this example is a type M red dwarf star on the main sequence. Assuming a type M9 red dwarf star is situated at the center of the sphere, it would have an approximate T_{eff} of 2300 K, a luminosity of 0.00015 $L_\odot$, a stellar radius of $R = 0.08R_\odot$, and a mass of 0.075 $M_\odot$ [4, Table 1]. In the MKS system, that leaves us with a luminosity of $L = 5.759 \times 10^{22}$W, a stellar radius of $R = 5.564 \times 10_7$m, and a stellar mass of 1.486×10^{29}kg.

For the purposes of analyzing this problem, only the radius and thickness of the spherical shell are variable. The thermal conductivity, output from the star, and solar mass are all assumed to be constant. Also, this ignores any material ejected from the star and deposited onto the interior surface of the sphere. The sphere is also assumed to be made of iron.

3.3 Desired surface conditions

The desired conditions on the exterior surface of the star are as follows: 1 G of gravity (here assumed to be $9.81\frac{m}{s^2}$) and a temperature of 298 K.

Naturally, an atmosphere would be required to sustain life on the surface, but the mass of such an atmosphere is neglected in the above equations.

4 Discussion

Periodic daylight on a Dyson Sphere may or may not be possible. If two smaller stars were orbiting on opposite sides, it may be possible to have periodic daylight. The two stars would have to be identical in gravitational attraction to the Dyson Sphere such that the barycenter of the total system does not move. Having two large gravitational bodies orbiting nearby, however, would likely place a strain on the sphere's structure by way of tidal forces.

If the spheral shell is rotating, it would have a lower gravitational acceleration at the equator and higher gravitational acceleration at the poles. To find this difference in gravity at the three different locations, it would be necessary to go through the steps of calculating the mass and size of the star-shell system and then applying some arbitrary spin to it.

The amount of material required to build such a structure would be immense and may require mining multiple star systems for raw materials. The materials used could vary. Comets are made of dust and ice. Asteroids are primarily nickel, iron, and silicates.

Only heat and gravity are taken into account in the above calculations. An atmosphere would be essential for sustaining any life, as well as a hydrologic system. Comets harvested for construction could be use to provide required water and gases.

A magnetic field would need to be generated to protect the atmosphere and any people who may be living on the surface from cosmic radiation.

As shown in Table 1, the smaller the outer radius of the shell, the less material is required to build it, regardless of the shell's thickness. This indicates that the shell requiring the least amount of material for construction can be described in Equation (12).

5 Conclusion

The smaller the Dyson sphere, the easier it would be to construct. A sphere similar to the on described in Equation (12) would make for the smallest possible shell, given a particular stellar radius, R. Because it is easier to construct a smaller shell than a larger one, future attempts to detect such objects may attempt to focus on areas with smaller stars. Given the sheer abundance of red dwarf stars, their small size, and their immense lifetimes, they would provide the most likely locations for a Dyson Sphere to be constructed.

6 Constants

Below are the relevant figures taken and used as constants in this paper [1, App. A].

Constant	Symbol	Value
Gravitational constant	G	6.673×10^{-11}N m^{2}kg^{-2}
Boltzmann constant	k	1.381^{-23}JK$^-1$
Solar mass	$M_\odot$	1.9891×10^{30}kg
Solar luminosity	$L_\odot$	3.839×10^{26}W
Solar radius	$R_\odot$	6.955×10^{8}m
Solar effective temp	$T_{eff,\odot}$	5777K

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